Research Statement — James R. Garrison

My research in **quantum condensed matter theory** is guided by a desire to investigate and understand the vast landscape of collective phenomena that can emerge from many-particle quantum mechanics. Whether it is electron transport in a solid or trapped ions interacting via long-range effective magnetic interactions, I am most interested in phenomena that can realistically be probed in near- to intermediate-term experiments.

My work builds upon knowledge from a number of areas, including atomic/molecular/optical (AMO) physics, high-performance scientific computing, quantum information, and computational complexity theory. My specific research interests include the dynamics of isolated quantum systems, zero-temperature phases of matter, and methods for simulating quantum mechanics. One broad theme that unifies these areas is the central importance of quantum entanglement.

Dynamics and thermalization of isolated quantum systems

Over the past few decades, experimental developments have provided platforms that are effectively isolated from the remainder of the universe on any relevant timescale, including, most notably, ultracold atoms with interactions tunable via Feshbach resonance [17, 18]. Such systems have enabled the direct study of nonequilibrium phenomena, including thermalization, localization [19–21], and particle transport [22].

One very fundamental problem in quantum statistical mechanics involves **whether—and how—an isolated quantum system will thermalize at long times** [23]. In quantum systems that do thermalize, the long-time expectation value of any "reasonable" operator will match its predicted value in the appropriate thermal ensemble. The **eigenstate thermalization hypothesis** (ETH) posits that this thermalization occurs at the level of each individual energy eigenstate [24–27]. My principal work in this field has involved identifying precisely which operators satisfy ETH, as well as the limits to the information contained in a single eigenstate [4]. Remarkably, a single eigenstate can contain information about energy densities—and therefore temperatures—far away from the energy density of the eigenstate. Follow-up work by others has provided evidence that a single eigenstate can even provide information regarding the critical behavior of a finite-temperature phase transition [28].

Notable exceptions to thermalization also exist, including integrable systems [29, 30], as well as those that exhibit **many-body localization** (MBL) due to a strong disorder potential [31–33]. One previous research project studied to what extent MBL can survive in a model system in which coupling is introduced between a disordered chain (which alone would exhibit localization) and a clean chain [5]. Another project demonstrated that the entanglement structure of eigenstates of noninteracting disorder models imply an emergent geometry in which the thermalization/localization status of the state is apparent [7]. Can MBL be used to store long-term quantum memories robustly? This question has both philosophical and practical relevance. Future work in this field will surely involve further probing the localization-to-thermalization transition, as well as making additional connections with experiment.

Another intriguing open question is whether there exist **phases of matter that are neither fully thermalized nor localized**—that is, phases which are non-ergodic despite entanglement that grows linearly with time [34–37]. Together with collaborators, I have provided numerical evidence for two qualitatively distinct types of states at overlapping energy densities in an extended Hubbard model, suggesting a clear violation of ETH in a non-integrable, translationally invariant system [6]. Recent work by others has suggested that a similar phenomenon, dubbed "quantum many-body scars," has revealed itself in the dynamics of a Rydberg atomic simulator [38]. This is a challenging field with important implications for isolated quantum systems and even quantum gravity, with analogy to the "no-hair theorem" for classical

black holes [39]. Without a doubt, progress with require a complex interplay among theory, experiment, and simulation.

I am also interested in the **short- to intermediate-time dynamics** of isolated quantum systems. One recent work considered Abelian anyons in one dimension following a quench, where we demonstrated asymmetry in particle transport and in information propagation, as studied via the out-of-time-ordered correlator [12]. Another recent work demonstrated a connection between a dynamical phase transition and the ground-state phase transition for systems near integrability [13], the consequences of which are accessible at prethermal (i.e., intermediate) time scales. Looking forward, I am most intrigued by simulation methods that may potentially allow the efficient and accurate probing of **transport dynamics and hydrodynamic properties at intermediate times**. Early progress has been made by using certain matrix-product-state methods, such as time-dependent variational principle [40] and density matrix truncation [41]. To what extent can classical simulation provide insight into intermediate-time dynamics, which are typically thought to be difficult or impossible to model on a classical computer? Exploring this question is sure to reveal deep insights into the dynamics of quantum matter, with implications for quantum information spreading and computational complexity theory.

Unconventional zero-temperature phases of quantum matter

The study of equilibrium phases is the traditional centerpiece of condensed matter physics. Along this front, I am most interested in unconventional, "quantum" phases of matter which exist at zero-temperature. These unconventional phases include those that lie beyond the standard symmetry-breaking and quasiparticle paradigms (both due to Landau), as well as phases with unusual entanglement properties, e.g. those whose ground state(s) violate the entanglement "area law" despite the existence of an energy gap between the ground state and first excited state.

One long-term goal of my research is to develop a sophisticated understanding of **ground-state phases in systems with long-range interactions**, specifically those that decay as a power law $1/r^\alpha$ with distance r. This goal is motivated by experimentalists' increasing ability to realize such interactions in AMO platforms, including interacting Rydberg atoms ($\alpha = 3$ electric dipole-dipole or $\alpha = 6$ van der Waals interactions) [42, 43], polar molecules ($\alpha = 3$ electric dipole-dipole interactions) [44], and trapped ions (tunable $0 \leq \alpha \leq 3$ phonon-mediated interactions) [45]. Such engineered platforms lie in stark contrast to the traditional setting of solid-state physics—i.e., metals—where long-range interactions are absent due to screening [46]. Despite these developments, much of physicists' thinking today—including even our best definition of what constitutes a distinct "gapped" phase of matter [47]—assumes only short-range, "local" interactions are present. Recent works have demonstrated that power-law decaying interactions enable many behaviors thought to be impossible in systems with only local interactions. For instance, it is possible to go through a topological phase transition without closing the gap if such interactions are present [48]. Other work has demonstrated the existence of a *gapped* phase in the one-dimensional long-range Ising model that violates the entanglement "area law" and exhibits correlation functions that decay as a polynomial [49]. Motivated by the latter development, I am currently working to better understand the phase diagram of the long-range Ising model in dimensions greater than one.[†] One exciting possibility is that long-range interactions will enable the existence of novel phases with no short-range counterpart. In any case, I expect that the study of long-range interactions will enable—and require—us, as physicists, to develop a more sophisticated framework for understanding quantum phases of matter.

[†] "Zero-temperature properties of the long-range transverse-field Ising model on the triangular lattice," Kevin Wang, Alexey V. Gorshkov, and James R. Garrison (in preparation).

I am also broadly interested in **unconventional phases of matter beyond the standard quasiparticle paradigm**. For instance, the "strange metal" phase which exists above the superconducting transition temperature in the copper-oxide, high-temperature superconductors is a striking example of a conducting phase that appears to violate Landau's Fermi liquid theory [50, 51]. I was involved with one of the earliest works that explicitly constructed a non-Fermi liquid phase distinct from a Luttinger liquid [2]. Another previous research project involved a study of variational wavefunctions for gapless spin liquids, motivated by experimental results on organic weak Mott insulators [3]. I expect that future progress in this area will require the convergence of insights gained from a variety of subfields, from new experimental probes, to improved numerical simulation and theoretical connections to quantum information.

Classical and quantum simulation of many-particle quantum mechanics

Developments in condensed matter physics have always relied on a close interplay between theory and experiment. In recent years, the simulation of quantum mechanics—most notably via numerical methods—has become a third pillar, enabling deeper physical understanding [52]. In some cases, simulation can take the role of an idealized experiment, providing exact results in systems that are unamenable to known analytic approaches. In others, simulation is more akin to theory, allowing us to better understand the effects of conjectured theoretical approximations and simplifications.

The classical simulation of quantum mechanics via numerical methods is a field with growing importance. **Exact diagonalization methods** allow us to query the behavior of model Hamiltonians at small system sizes, including ground states, excited states, thermal states, and dynamics [53, 54]. **Monte Carlo techniques** include both exact (e.g. the stochastic series expansion [55]) and approximate (e.g. variational Monte Carlo [56]) methods, and can provide invaluable insight in situations where the "sign problem" [57] is absent. **Tensor-network methods** provide a conceptual framework for simulating states with limited entanglement. Most notably, the matrix-product-state ansatz [58, 59] has revolutionized our ability to simulate (quasi-)one-dimensional systems through such methods as the ground-state density-matrix renormalization group (DMRG) [60], the time-evolving block decimation (TEBD) [61], and the time-dependent variational principle (TDVP) [62, 63]. I have personally implemented each of the aforementioned methods. My software development knowledge draws upon decades of experience, including time spent professionally as a full-time software engineer for a #1-ranked computer science department.

I expect that my research group will be at the forefront of using numerical techniques to make progress in the research areas described in the previous sections, and I expect that we will be equally invested in improving the techniques and algorithms themselves. Looking forward, more advanced tensor-network methods, including the multi-scale entanglement renormalization ansatz (MERA) [64] and projected entangled-pair states (PEPS) [65, 66], have the potential to bring the power of simulation methods to gapless systems, as well as systems in dimension greater than one. Likewise, improved Monte Carlo methods will enable the simulation of systems that are currently intractable, just as they have in the past [52].

Of course, generic and exact simulation of quantum mechanics appears to require resources that scale exponentially with system size, thus rendering every classical simulation method intractable beyond a certain point. It was this realization that prompted Richard Feynman, in 1981, to propose the development of **quantum computers** as devices which can harness the power of quantum mechanics for simulating quantum mechanics [67]. Nowadays, such quantum simulation methods have both analog and digital variants. **Analog quantum simulators** are highly controlled and tuned quantum systems designed to model a desired Hamiltonian. One example of such a system is ultracold atoms in an optical lattice, in which experimentalists demonstrated in 2002 the superfluid to Mott insulator phase transition in the Bose-Hubbard model [68]. Along this front, one of my collaborations has developed a method for state preparation of a Chern

insulator in a cold atomic gas, based on a continuous version of MERA together with a long-range, engineered Hamiltonian [9]. I am also interested in **digital quantum simulation**, i.e., methods for simulating quantum mechanics on coherently controlled, gate-based quantum computers. My work with collaborators includes the best algorithm known to date for systems with power-law decaying interactions [10]. I have also worked to improve methods for simulating fermionic quantum field theory on a quantum computer [16]. Looking forward, I expect that **hybrid algorithms**, which combine aspects of both classical and quantum simulation, will be important on early quantum computers—so-called noisy, intermediate-scale quantum ("NISQ") devices [69].

Quantum entanglement in many-body systems

Quantum entanglement is one broad theme that unites each of the above research areas. Our growing understanding of—and ability to calculate—the entanglement structure of many-body quantum systems has provided remarkable insight into their properties. It has allowed us to diagnose and characterize both ground state phases [70] and states at finite energy density [71]. Understanding the connection between entanglement and matrix-product states has given us the ability to classically simulate states with limited entanglement [72]. Conversely, for a quantum algorithm to result in a quantum speed-up, it must generate sufficient entanglement; otherwise it could have been efficiently simulated on a classical computer.

Entanglement appears in a number of other contexts as well. For instance, quantum speed limits known as "**Lieb-Robinson bounds**" place theoretical restrictions on the rate at which information can propagate in a non-relativistic quantum system [73]. These bounds have profound implications both for the dynamical spreading of entanglement and for the amount of entanglement permissible in a system's ground state (e.g., the so-called "area law") [74]. Together with collaborators, my work on this front has explored Lieb-Robinson bounds on multipartite correlation functions [8] and provided a tighter light cone for systems with long-range interactions that decay as a power law [10].

Entanglement is at the heart of quantum condensed matter physics. Already, much has been gained from the calculation of relatively simple quantities, including the von Neumann and Rényi entanglement entropies, as well as the spectra of bipartite cuts [75]. Looking forward, I expect that developments in quantum information theory will continue to provide new ways of probing and understanding the structure of entanglement in quantum matter, leading us to a deeper physical intuition.

Publications and preprints

[1] "A novel hybrid simulation for study of multiscale phenomena," P. E. Krouskop, J. Garrison, P. C. Gedeon, and J. D. Madura, *Molecular Simulation* **32**, 825 (2006).

[2] "Non-Fermi-liquid d-wave metal phase of strongly interacting electrons," H.-C. Jiang, M. S. Block, R. V. Mishmash, J. R. Garrison, D. N. Sheng, O. I. Motrunich, and M. P. A. Fisher, *Nature* **493**, 39 (2013) [arXiv:1207.6608].

[3] "Theory of a Competitive Spin Liquid State for Weak Mott Insulators on the Triangular Lattice," Ryan V. Mishmash, James R. Garrison, Samuel Bieri, and Cenke Xu, *Phys. Rev. Lett.* **111**, 157203 (2013) [arXiv:1307.0829].

[4] "Does a single eigenstate encode the full Hamiltonian?" James R. Garrison and Tarun Grover, *Phys. Rev. X* **8**, 021026 (2018) [arXiv:1503.00729].

[5] "Many-body localization in the presence of a small bath," Katharine Hyatt, James R. Garrison, Andrew C. Potter, and Bela Bauer, *Phys. Rev. B* **95**, 035132 (2017) [arXiv:1601.07184].

[6] "Partial breakdown of quantum thermalization in a Hubbard-like model," James R. Garrison, Ryan V. Mishmash, and Matthew P. A. Fisher, *Phys. Rev. B* **95**, 054204 (2017) [arXiv:1606.05650].

[7] "Extracting entanglement geometry from quantum states," Katharine Hyatt, James R. Garrison, and Bela Bauer, *Phys. Rev. Lett.* **119**, 140502 (2017) [arXiv:1704.01974].

[8] "Lieb-Robinson bounds on n-partite connected correlation functions," Minh Cong Tran, James R. Garrison, Zhe-Xuan Gong, and Alexey V. Gorshkov, *Phys. Rev. A* **96**, 052334 (2017) [arXiv:1705.04355].

[9] "Scale-Invariant Continuous Entanglement Renormalization of a Chern Insulator," Su-Kuan Chu, Guanyu Zhu, James R. Garrison, Zachary Eldredge, Ana Valdés Curiel, Przemyslaw Bienias, I. B. Spielman, and Alexey V. Gorshkov, *Phys. Rev. Lett.* **122**, 120502 (2019) [arXiv:1807.11486].

[10] "Locality and digital quantum simulation of power-law interactions," Minh C. Tran, Andrew Y. Guo, Yuan Su, James R. Garrison, Zachary Eldredge, Michael Foss-Feig, Andrew M. Childs, and Alexey V. Gorshkov, *Phys. Rev. X* **9**, 031006 (2019) [arXiv:1808.05225].

[11] "Unitary entanglement construction in hierarchical networks," Aniruddha Bapat, Zachary Eldredge, James R. Garrison, Abhinav Desphande, Frederic T. Chong, and Alexey V. Gorshkov, *Phys. Rev. A* **98**, 062328 (2018) [arXiv:1808.07876].

[12] "Asymmetric Particle Transport and Light-Cone Dynamics Induced by Anyonic Statistics," Fangli Liu, James R. Garrison, Dong-Ling Deng, Zhe-Xuan Gong, and Alexey V. Gorshkov, *Phys. Rev. Lett.* **121**, 250404 (2018) [arXiv:1809.02614].

[13] "Probing Ground-State Phase Transitions through Quench Dynamics," Paraj Titum, Joseph T. Iosue, James R. Garrison, Alexey V. Gorshkov, and Zhe-Xuan Gong, *Phys. Rev. Lett.* **123**, 115701 (2019) [arXiv:1809.06377].

[14] "Circuit complexity across a topological phase transition," Fangli Liu, Seth Whitsitt, Jonathan B. Curtis, Rex Lundgren, Paraj Titum, Zhi-Cheng Yang, James R. Garrison, and Alexey V. Gorshkov, *Phys. Rev. Research* **2**, 013323 (2020) [arXiv:1902.10720].

[15] "Entanglement bounds on the performance of quantum computing architectures," Zachary Eldredge, Leo Zhou, Aniruddha Bapat, James R. Garrison, Abhinav Deshpande, Frederic T. Chong, and Alexey V. Gorshkov, *Phys. Rev. Research* **2**, 033316 (2020) [arXiv:1908.04802].

[16] "Site-by-site quantum state preparation algorithm for preparing vacua of fermionic lattice field theories," Ali Hamed Moosavian, James R. Garrison, and Stephen P. Jordan [arXiv:1911.03505].

References

[17] S. Inouye, M. R. Andrews, J. Stenger, H.-J. Miesner, D. M. Stamper-Kurn, and W. Ketterle, "Observation of Feshbach resonances in a Bose–Einstein condensate," Nature **392**, 151 (1998).

[18] E. A. Donley, N. R. Claussen, S. L. Cornish, J. L. Roberts, E. A. Cornell, and C. E. Wieman, "Dynamics of collapsing and exploding Bose–Einstein condensates," Nature **412**, 295 (2001).

[19] J. Smith, A. Lee, P. Richerme, B. Neyenhuis, P. W. Hess, P. Hauke, M. Heyl, D. A. Huse, and C. Monroe, "Many-body localization in a quantum simulator with programmable random disorder," Nature Physics **12**, 907 (2016), arXiv:1508.07026 [quant-ph] .

[20] M. Schreiber, S. S. Hodgman, P. Bordia, H. P. Lüschen, M. H. Fischer, R. Vosk, E. Altman, U. Schneider, and I. Bloch, "Observation of many-body localization of interacting fermions in a quasirandom optical lattice," Science **349**, 842 (2015), arXiv:1501.05661 [cond-mat.quant-gas] .

[21] S. S. Kondov, W. R. McGehee, W. Xu, and B. DeMarco, "Disorder-induced localization in a strongly correlated atomic hubbard gas," Phys. Rev. Lett. **114**, 083002 (2015), arXiv:1305.6072 .

[22] C.-C. Chien, S. Peotta, and M. Di Ventra, "Quantum transport in ultracold atoms," Nature Physics **11**, 998 (2015).

[23] L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, "From quantum chaos and eigenstate thermalization to statistical mechanics and thermodynamics," Advances in Physics **65**, 239 (2016), arXiv:1509.06411 [cond-mat.stat-mech] .

[24] J. M. Deutsch, "Quantum statistical mechanics in a closed system," Phys. Rev. A **43**, 2046 (1991).

[25] M. Srednicki, "Chaos and quantum thermalization," Phys. Rev. E **50**, 888 (1994).

[26] M. Srednicki, "The approach to thermal equilibrium in quantized chaotic systems," Journal of Physics A: Mathematical and General **32**, 1163 (1999), cond-mat/9809360 .

[27] M. Rigol, V. Dunjko, and M. Olshanii, "Thermalization and its mechanism for generic isolated quantum systems," Nature **452**, 854 (2008), arXiv:0708.1324 [cond-mat.stat-mech] .

[28] K. R. Fratus and S. V. Truong, "Does a Single Eigenstate of a Hamiltonian Encode the Critical Behaviour of its Finite-Temperature Phase Transition?" arXiv:1810.11092 [cond-mat] (2018), arXiv:1810.11092 [cond-mat] .

[29] P. Calabrese and J. Cardy, "Entanglement and correlation functions following a local quench: a conformal field theory approach," Journal of Statistical Mechanics: Theory and Experiment **10**, 4 (2007), arXiv:0708.3750 [cond-mat.stat-mech] .

[30] M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, "Relaxation in a Completely Integrable Many-Body Quantum System: An Ab Initio Study of the Dynamics of the Highly Excited States of 1D Lattice Hard-Core Bosons," Phys. Rev. Lett. **98**, 050405 (2007), cond-mat/0604476 .

[31] V. Oganesyan and D. A. Huse, "Localization of interacting fermions at high temperature," Phys. Rev. B **75**, 155111 (2007), cond-mat/0610854 .

[32] A. Pal and D. A. Huse, "Many-body localization phase transition," Phys. Rev. B **82**, 174411 (2010), arXiv:1010.1992 [cond-mat.dis-nn] .

[33] D. J. Luitz, N. Laflorencie, and F. Alet, "Many-body localization edge in the random-field Heisenberg chain," Phys. Rev. B **91**, 081103 (2015), arXiv:1411.0660 [cond-mat.dis-nn] .

[34] T. Grover and M. P. A. Fisher, "Quantum disentangled liquids," Journal of Statistical Mechanics: Theory and Experiment **10**, 10010 (2014), arXiv:1307.2288 [cond-mat.str-el] .

[35] M. Schiulaz and M. Müller, "Ideal quantum glass transitions: Many-body localization without quenched disorder," in *American Institute of Physics Conference Series*, American Institute of Physics Conference Series, Vol. 1610 (2014) p. 11, arXiv:1309.1082 [cond-mat.dis-nn] .

[36] J. M. Hickey, S. Genway, and J. P. Garrahan, "Signatures of many-body localisation in a system without disorder and the relation to a glass transition," Journal of Statistical Mechanics: Theory and Experiment **2016**, 054047 (2016).

[37] N. Y. Yao, C. R. Laumann, J. I. Cirac, M. D. Lukin, and J. E. Moore, "Quasi-many-body localization in translation-invariant systems," Phys. Rev. Lett. **117**, 240601 (2016), arXiv:1410.7407 [cond-mat.dis-nn] .

[38] S. Choi, C. J. Turner, H. Pichler, W. W. Ho, A. A. Michailidis, Z. Papić, M. Serbyn, M. D. Lukin, and D. A. Abanin, "Emergent SU(2) dynamics and perfect quantum many-body scars," Phys. Rev. Lett. **122**, 220603 (2019), arXiv:1812.05561 .

[39] S. Khlebnikov and M. Kruczenski, "Thermalization of isolated quantum systems," ArXiv e-prints (2013), arXiv:1312.4612 [cond-mat.stat-mech] .

[40] E. Leviatan, F. Pollmann, J. H. Bardarson, D. A. Huse, and E. Altman, "Quantum thermalization dynamics with Matrix-Product States," ArXiv e-prints (2017), arXiv:1702.08894 [cond-mat.stat-mech] .

[41] C. D. White, M. Zaletel, R. S. K. Mong, and G. Refael, "Quantum dynamics of thermalizing systems," Phys. Rev. B **97** (2018), 10.1103/physrevb.97.035127.

[42] D. Jaksch, J. I. Cirac, P. Zoller, S. L. Rolston, R. Cote, and M. D. Lukin, "Fast quantum gates for neutral atoms," Phys. Rev. Lett. **85**, 2208 (2000), arXiv:quant-ph/0004038 .

[43] H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, and H. P. Büchler, "A Rydberg Quantum Simulator," Nature Physics **6**, 382 (2010), arXiv:0907.1657 .

[44] B. Yan, S. A. Moses, B. Gadway, J. P. Covey, K. R. A. Hazzard, A. M. Rey, D. S. Jin, and J. Ye, "Observation of dipolar spin-exchange interactions with lattice-confined polar molecules," Nature **501**, 521 (2013).

[45] J. W. Britton, B. C. Sawyer, A. C. Keith, C.-C. J. Wang, J. K. Freericks, H. Uys, M. J. Biercuk, and J. J. Bollinger, "Engineered two-dimensional Ising interactions in a trapped-ion quantum simulator with hundreds of spins," Nature **484**, 489 (2012), arXiv:1204.5789 [quant-ph] .

[46] N. Ashcroft and N. Mermin, *Solid State Physics*, HRW International Editions (Holt, Rinehart and Winston, 1976).

[47] X. Chen, Z.-C. Gu, and X.-G. Wen, "Local unitary transformation, long-range quantum entanglement, wave function renormalization, and topological order," Phys. Rev. B **82**, 155138 (2010), arXiv:1004.3835 [cond-mat.str-el] .

[48] D. Vodola, L. Lepori, E. Ercolessi, A. V. Gorshkov, and G. Pupillo, "Kitaev Chains with Long-Range Pairing," Phys. Rev. Lett. **113**, 156402 (2014), arXiv:1405.5440 [cond-mat.str-el] .

[49] T. Koffel, M. Lewenstein, and L. Tagliacozzo, "Entanglement Entropy for the Long-Range Ising Chain in a Transverse Field," Phys. Rev. Lett. **109**, 267203 (2012), arXiv:1207.3957 [cond-mat.str-el] .

[50] P. A. Lee, N. Nagaosa, and X.-G. Wen, "Doping a mott insulator: Physics of high-temperature superconductivity," Rev. Mod. Phys. **78**, 17 (2006).

[51] G. S. Boebinger, "An Abnormal Normal State," Science **323**, 590 (2009).

[52] D. Landau and K. Binder, *A Guide to Monte Carlo Simulations in Statistical Physics* (Cambridge University Press, USA, 2005).

[53] J. C. Bonner and M. E. Fisher, "Linear Magnetic Chains with Anisotropic Coupling," Phys. Rev. **135**, A640 (1964).

[54] A. Weiße and H. Fehske, "Exact Diagonalization Techniques," in *Computational Many-Particle Physics*, Lecture Notes in Physics, Vol. 739, edited by H. Fehske, R. Schneider, and A. Weiße (Springer Berlin Heidelberg, 2008) p. 529.

[55] A. W. Sandvik, "Stochastic series expansion method for quantum Ising models with arbitrary interactions," Phys. Rev. E **68**, 056701 (2003), cond-mat/0303597 .

[56] D. Ceperley, G. V. Chester, and M. H. Kalos, "Monte carlo simulation of a many-fermion study," Phys. Rev. B **16**, 3081 (1977).

[57] S. Bravyi, D. P. DiVincenzo, R. I. Oliveira, and B. M. Terhal, "The Complexity of Stoquastic Local Hamiltonian Problems," (2006).

[58] S. Östlund and S. Rommer, "Thermodynamic Limit of Density Matrix Renormalization," Phys. Rev. Lett. **75**, 3537 (1995), cond-mat/9503107 .

[59] U. Schollwöck, "The density-matrix renormalization group in the age of matrix product states," Annals of Physics **326**, 96 (2011), arXiv:1008.3477 [cond-mat.str-el] .

[60] S. R. White, "Density matrix formulation for quantum renormalization groups," Phys. Rev. Lett. **69**, 2863 (1992).

[61] G. Vidal, "Efficient simulation of one-dimensional quantum many-body systems," Phys. Rev. Lett. **93**, 040502 (2004), arXiv:quant-ph/0310089 .

[62] J. Haegeman, J. I. Cirac, T. J. Osborne, I. Pižorn, H. Verschelde, and F. Verstraete, "Time-Dependent Variational Principle for Quantum Lattices," Phys. Rev. Lett. **107**, 070601 (2011), arXiv:1103.0936 [cond-mat.str-el] .

[63] J. Haegeman, C. Lubich, I. Oseledets, B. Vandereycken, and F. Verstraete, "Unifying time evolution and optimization with matrix product states," ArXiv e-prints (2014), arXiv:1408.5056 [quant-ph] .

[64] G. Vidal, "Class of Quantum Many-Body States That Can Be Efficiently Simulated," Phys. Rev. Lett. **101**, 110501 (2008), quant-ph/0610099 .

[65] F. Verstraete and J. I. Cirac, "Renormalization algorithms for Quantum-Many Body Systems in two and higher dimensions," arXiv:cond-mat/0407066 (2004), arXiv:cond-mat/0407066 .

[66] C. V. Kraus, N. Schuch, F. Verstraete, and J. I. Cirac, "Fermionic Projected Entangled Pair States," Phys. Rev. A **81**, 052338 (2010), arXiv:0904.4667 .

[67] R. P. Feynman, "Simulating physics with computers," International Journal of Theoretical Physics **21**, 467.

[68] M. Greiner, O. Mandel, T. Esslinger, T. W. Hansch, and I. Bloch, "Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms," Nature **415**, 39 (2002).

[69] J. Preskill, "Quantum Computing in the NISQ era and beyond," arXiv:1801.00862 [cond-mat, physics:quant-ph] (2018), 10.22331/q-2018-08-06-79, arXiv:1801.00862 [cond-mat, physics:quant-ph] .

[70] P. Calabrese and J. Cardy, "Entanglement entropy and quantum field theory," Journal of Statistical Mechanics: Theory and Experiment **6**, 2 (2004), hep-th/0405152 .

[71] B. Bauer and C. Nayak, "Area laws in a many-body localized state and its implications for topological order," Journal of Statistical Mechanics: Theory and Experiment **9**, 09005 (2013), arXiv:1306.5753 [cond-mat.dis-nn] .

[72] G. Vidal, "Efficient Classical Simulation of Slightly Entangled Quantum Computations," Phys. Rev. Lett. **91** (2003), 10.1103/PhysRevLett.91.147902.

[73] E. H. Lieb and D. W. Robinson, "The finite group velocity of quantum spin systems," Comm. Math. Phys. **28**, 251 (1972).

[74] M. B. Hastings, "An area law for one-dimensional quantum systems," Journal of Statistical Mechanics: Theory and Experiment **8**, 24 (2007), arXiv:0705.2024 [quant-ph] .

[75] H. Li and F. D. M. Haldane, "Entanglement Spectrum as a Generalization of Entanglement Entropy: Identification of Topological Order in Non-Abelian Fractional Quantum Hall Effect States," Phys. Rev. Lett. **101**, 010504 (2008), arXiv:0805.0332 .